Programming TLS in Isabelle/HOL

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Isabelle Workshop 2014

supported by SNF

Isabelle/HOL as a programming language

Isabelle/HOL as a programming language

```
datatype color = R | B
datatype ('a, 'b) rbt
  = Empty
  | Branch color "('a, 'b) rbt" 'a 'b "('a, 'b) rbt"
```

datatypes

Isabelle/HOL as a programming language

```
fun merge :: "('b ⇒ 'a) ⇒ 'b list ⇒ 'b list ⇒ 'b list" where
  "merge key (a#as) (b#bs) = (if key a > key b
    then b # merge key (a#as) bs
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| "merge key [] bs = bs"
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```

recursive functions

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partial_function (tailrec) fixpoint :: "('a ⇒ 'a) ⇒ 'a ⇒ 'a"
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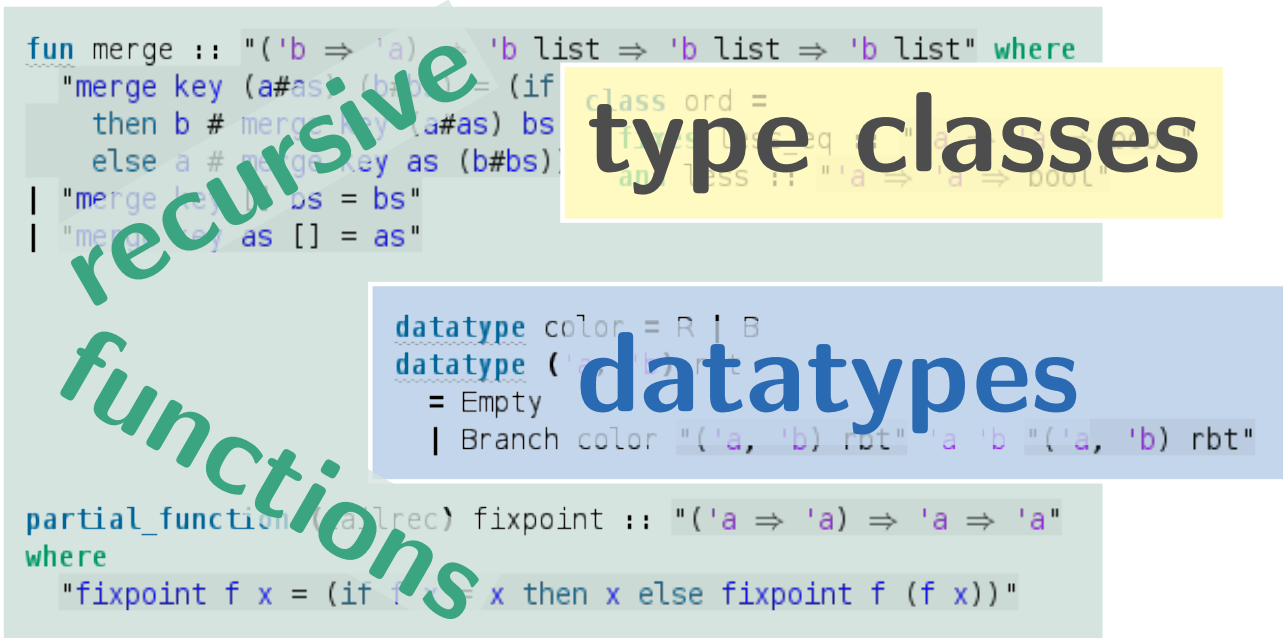

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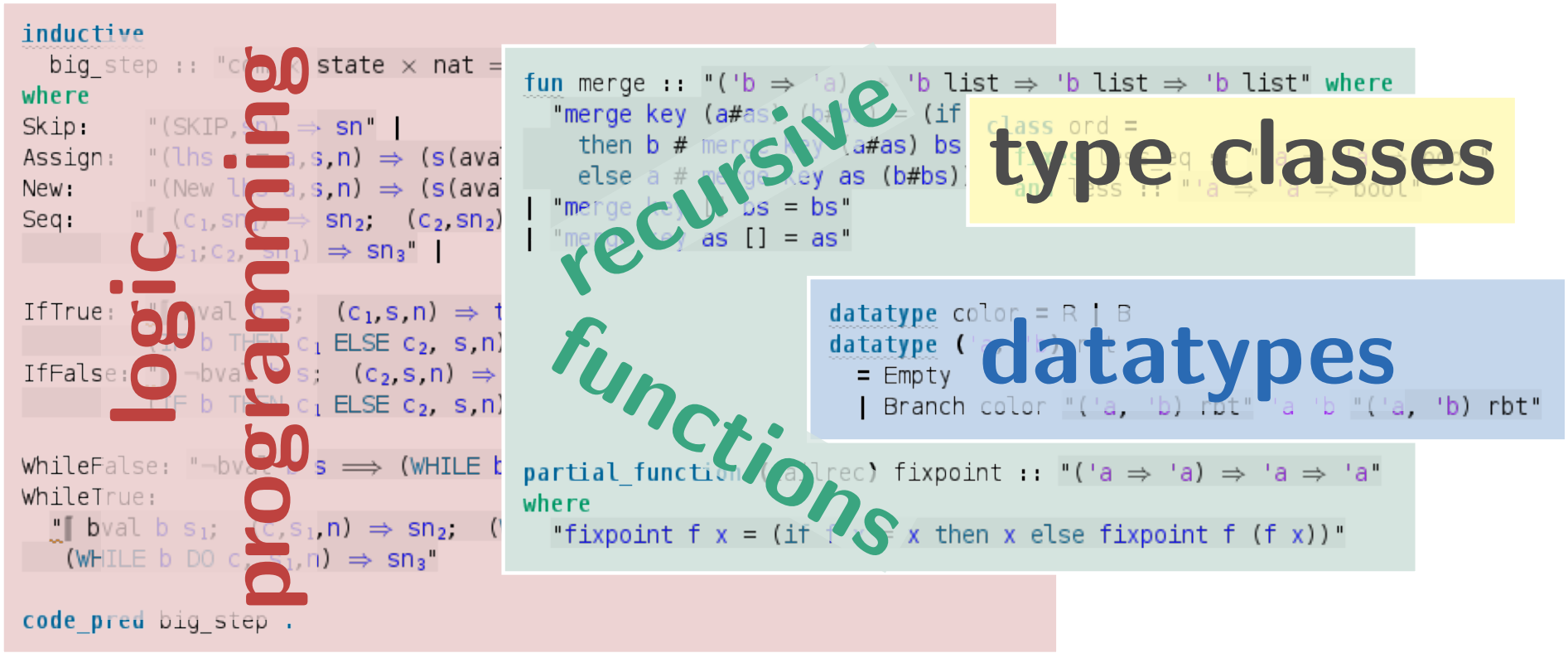


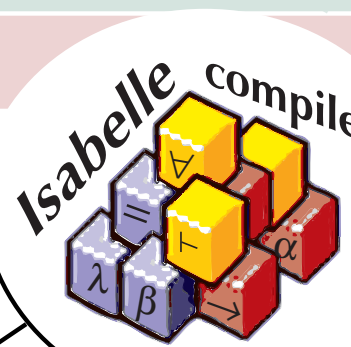
Isabelle/HOL as a programming language

logic programming

recursive functions

type classes

datatypes



Standard ML

Caml

Haskell

Scala

Isabelle/HOL as a programming language

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where
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New:     "(New lhs a,s) ⇒ (s(ava
Seq:     "⟦ (c₁,s₁) ⇒ s₂;  (c₂,s₂)
           (c₁;c₂, s₁) ⇒ s₃" |

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code_pred big_step .
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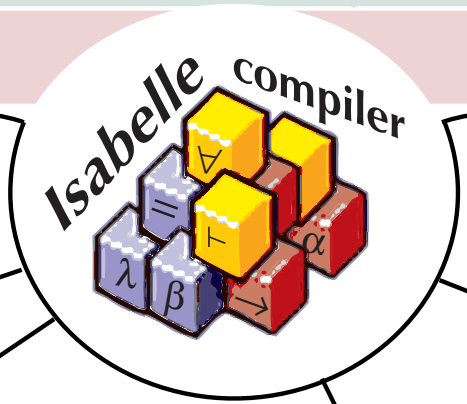

Quickcheck
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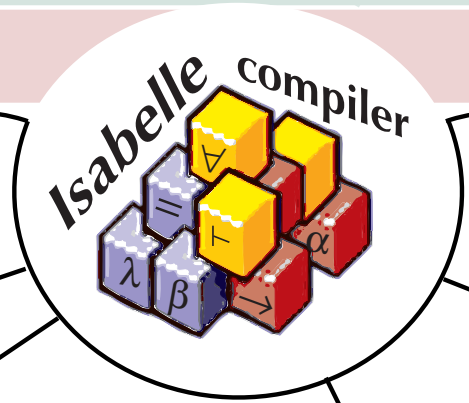

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CAVA, CeTA
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Case Study: Transport Layer Security (TLS)

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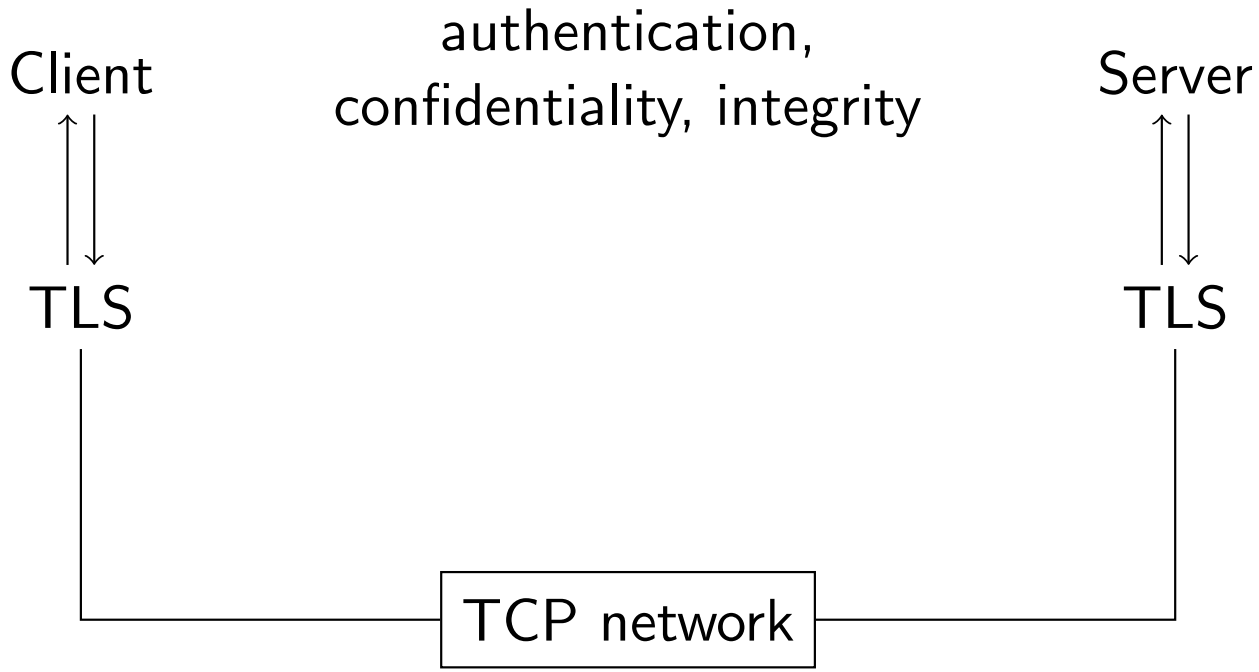

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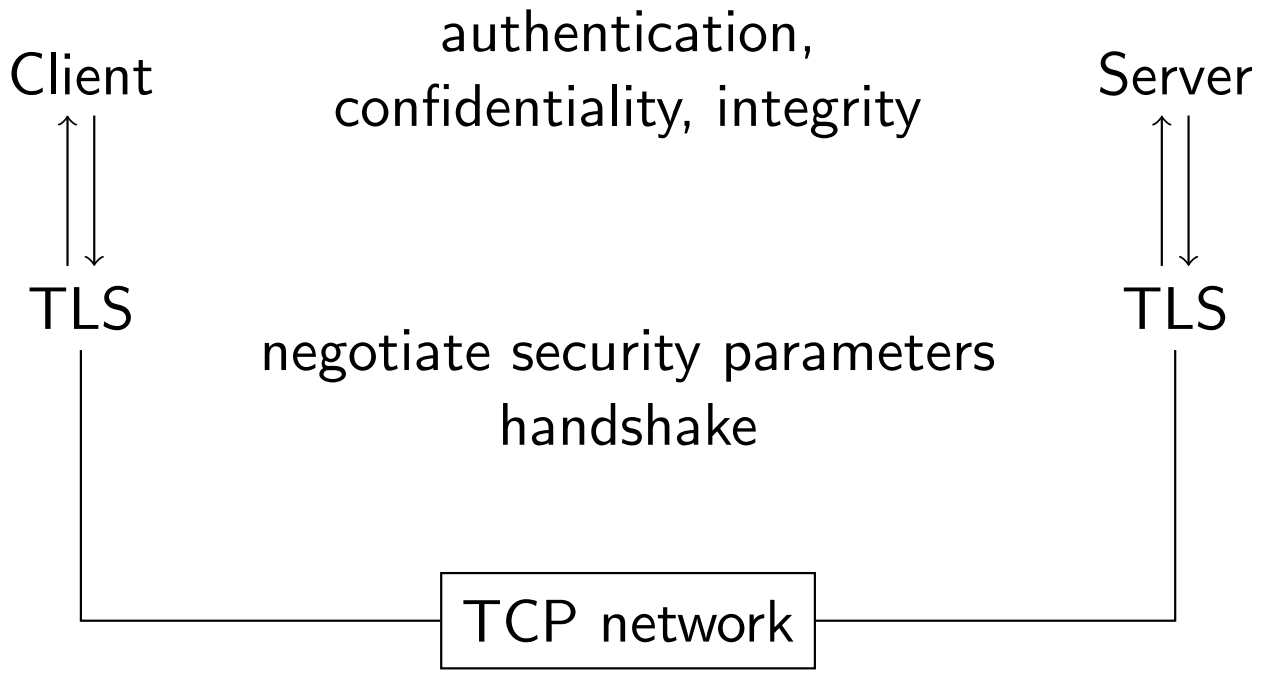

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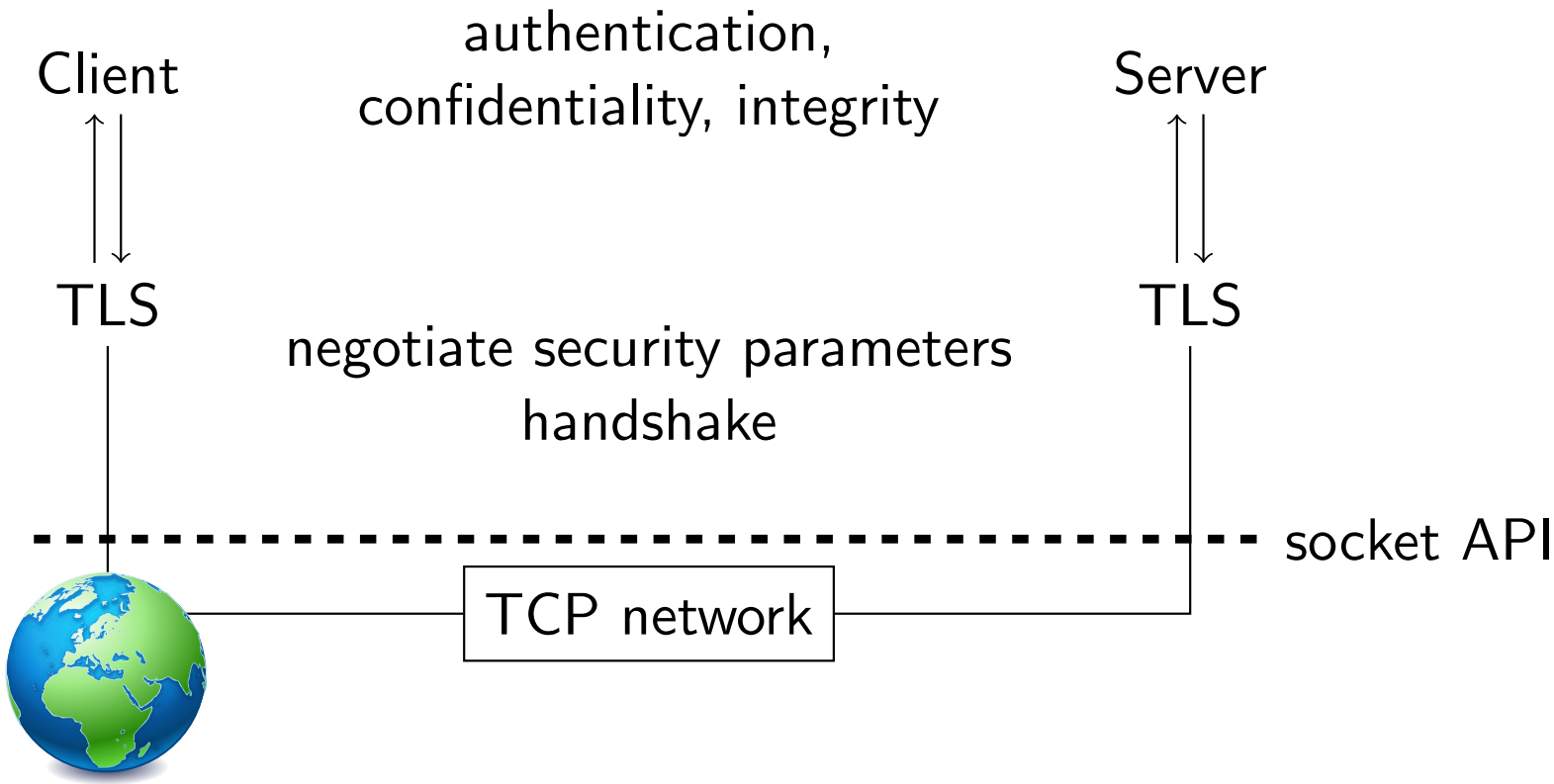

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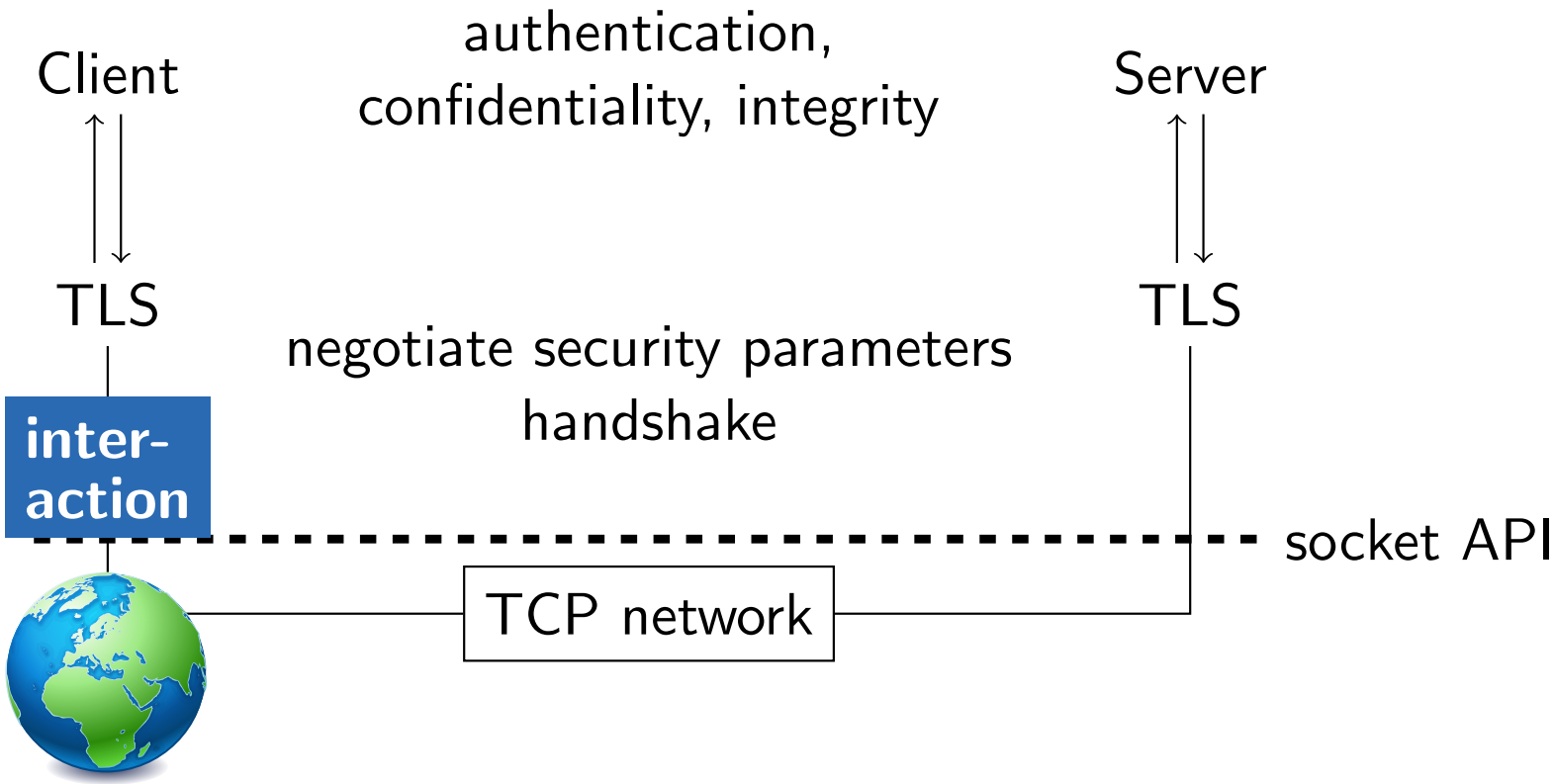

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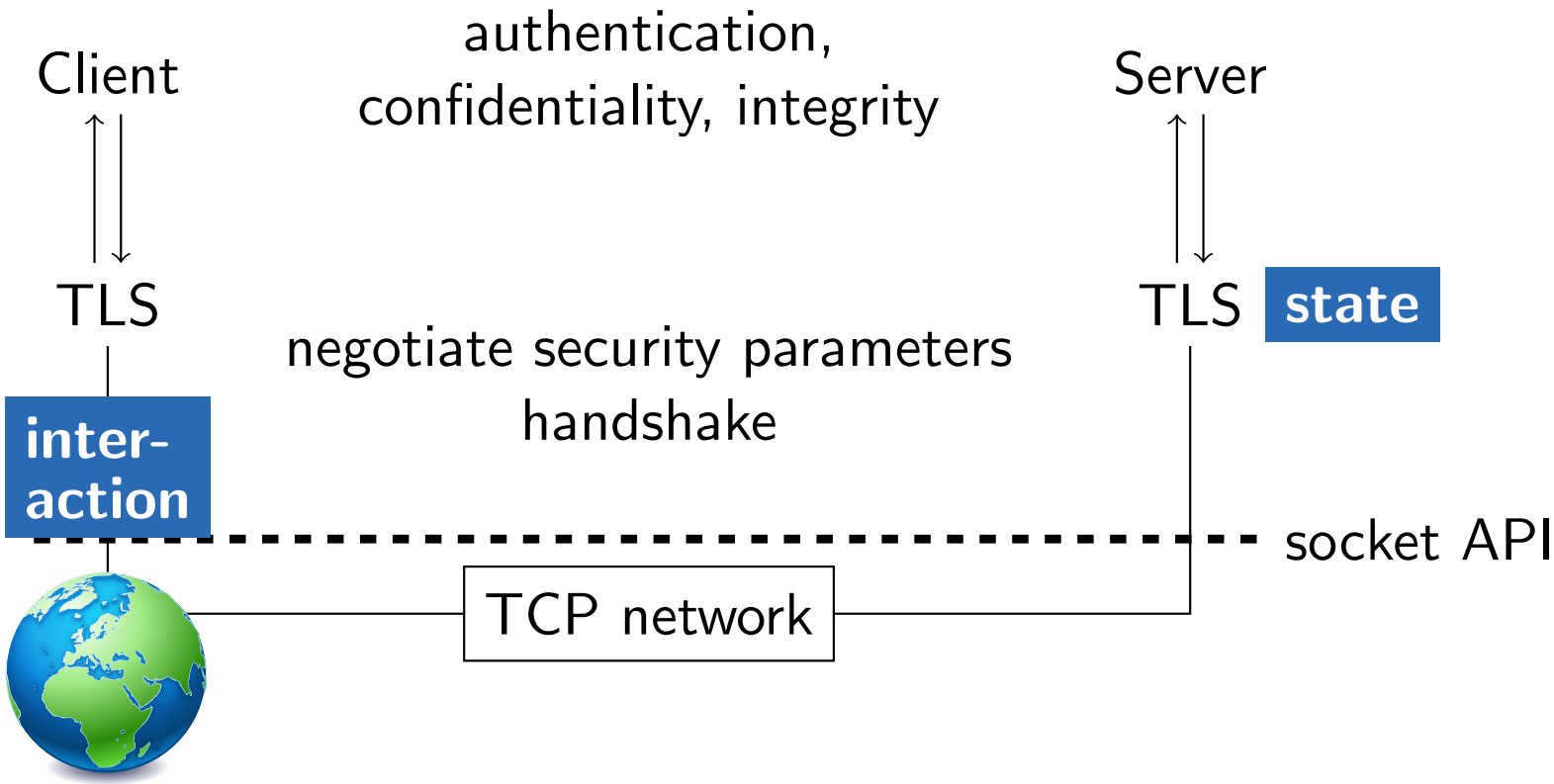

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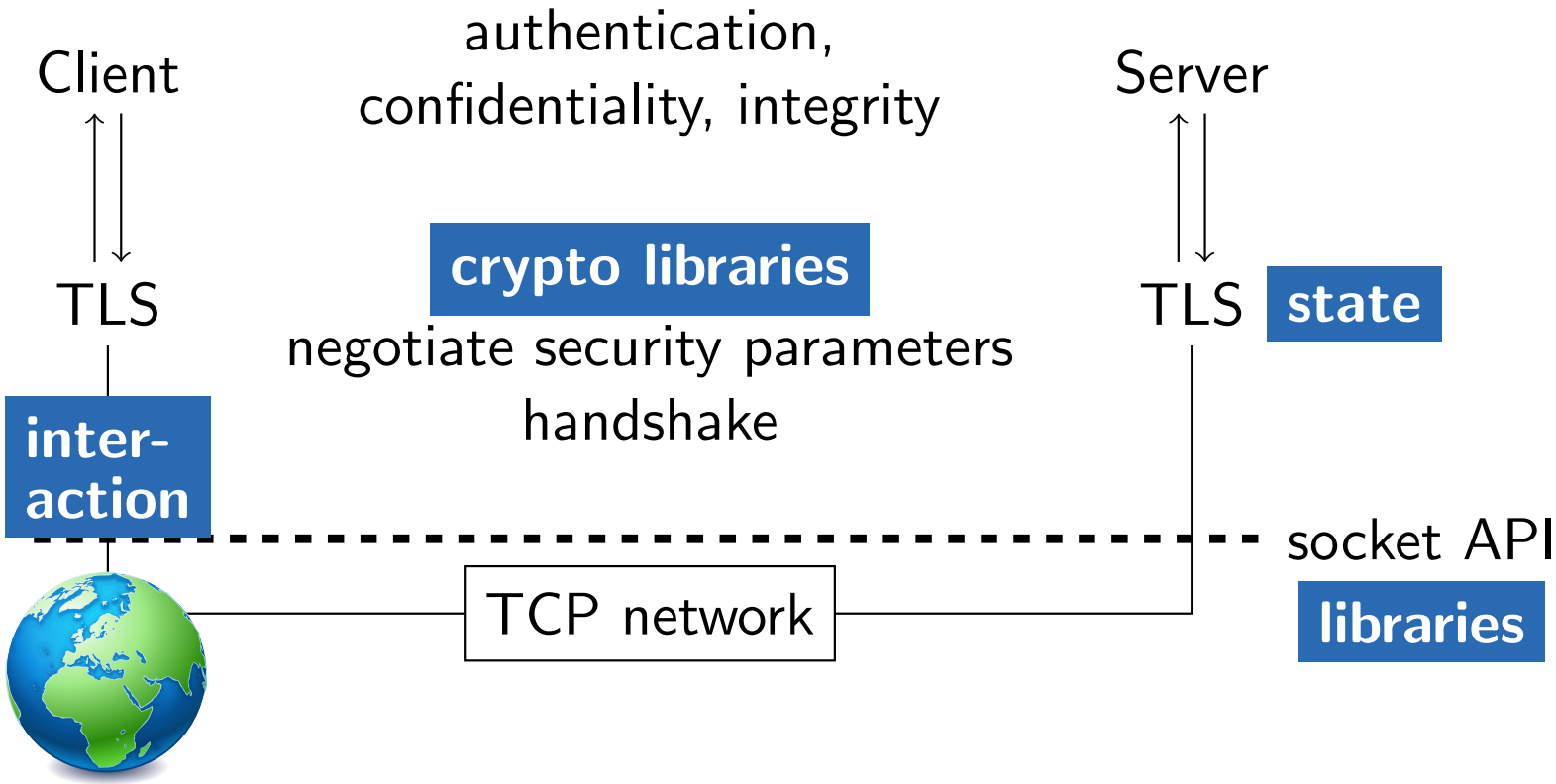
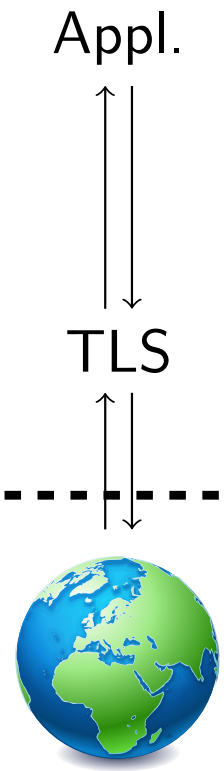


Interactive values



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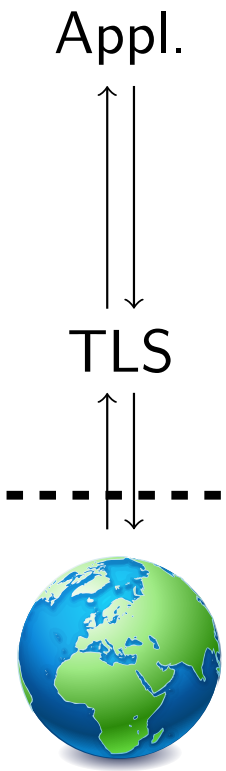

- cannot model socket API with ordinary HOL functions
- make interaction explicit in HOL

codatatype (α, o, ι) *resumption* =

Pure (*result*: α)
| *IO* (*output*: o) (*continuation*: $\iota \Rightarrow (\alpha, o, \iota)$ *resumption*)

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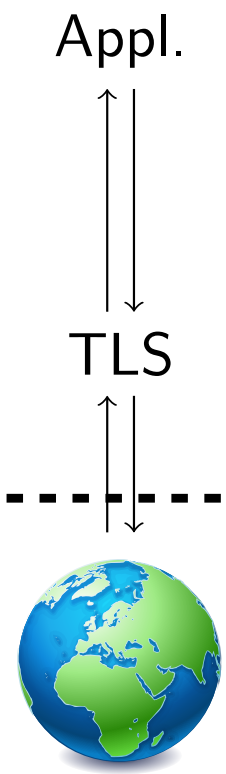

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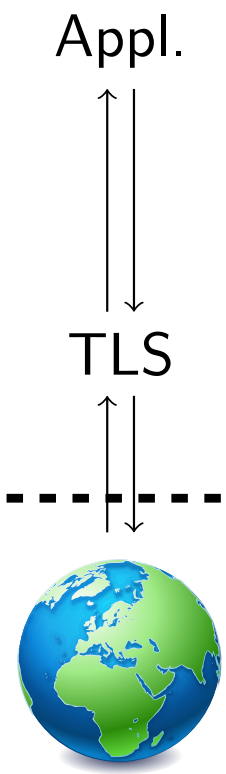

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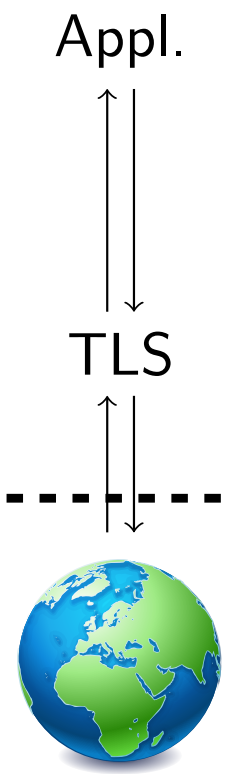
- add monad structure and setup for **partial-function**

stdin = *IO StdIn* (λ*in*. *case in of Receive s* ⇒ *Pure s* | _ ⇒ *Fail*)
stdout s = *IO* (*StdOut s*) (λ*in*. *case in of Ack* ⇒ *Pure* () | _ ⇒ *Fail*)

definition *hello* where *hello* = *do* { *s* ← *stdin*; *stdout* (*"Hello, " @ s*) }

Interactive values



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probabilistic interactive values

- cryptography requires a good source of randomness
- combine resumption with reader monad over coin flips
- ensures that randomness is used at most once

- add monad structure and setup for **partial-function**

$$stdin = IO\ StdIn\ (\lambda in.\ case\ in\ of\ Receive\ s \Rightarrow Pure\ s \mid \_ \Rightarrow Fail)$$
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Semantics and compilation of interactive values

Trace semantics

codatatype (α, ε) *trace* $= [\,]_{(\alpha\ \mathit{option})} \mid \mathit{TCons}\ \varepsilon\ ((\alpha, \varepsilon)\ \mathit{trace})$

$$\begin{aligned}
\mathit{traces}\ \mathit{Fail} &= \{\,[\,]_{\mathit{None}}\,\} \\
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Compilation via an interpreter into the IO monad

typedecl 🌍

typedef α *IO* $\cong$ 🌍 $\rightharpoonup \alpha \times$ 🌍

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consts *stdin-impl* :: *string IO*

stdout-impl :: *string* $\Rightarrow$ *unit IO*

$$
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Interpreter separates execution and proving

- ⊕ Application-specific models of the environment possible
- ⊕ Efficient: interactions are interpreted, computation is native
- ⊖ Generated code is hardly readable

Foreign function interface

Import and re-use library functions

- crypto algorithms
- I/O, socket API
- parser combinators

Foreign function interface

Import and re-use library functions

1. Declare unspecified types and constants
   typedecl *byte-string*
   consts *hash-sha1* :: *byte-string* ⇒ *byte-string*
2. Adapt code serialisation with **code-printing**

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Types for data exchange

- *unit*, *bool*, *integer*, *char*, _ *option*, _ *list*, *String.literal*
- ⊕ machine arithmetic *uint8*, *uint16*, . . . — Native Word in the AFP
- ⊖ string buffers — convert to *string*

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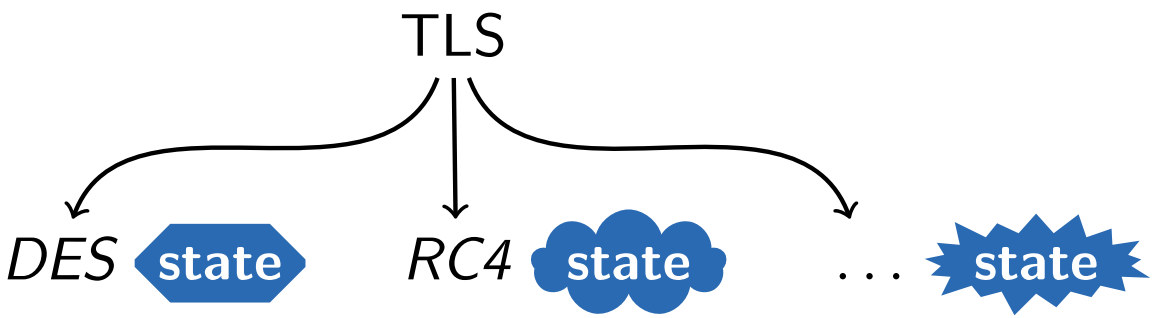
Existential state types

TLS state stores varying ciphersuites



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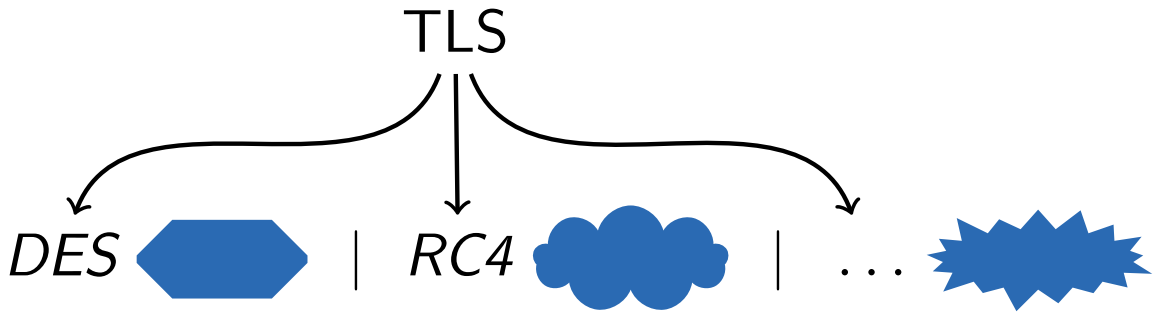

datatype *cipher* =

$$encrypt :: cipher \Rightarrow string \Rightarrow (cipher \times string)$$
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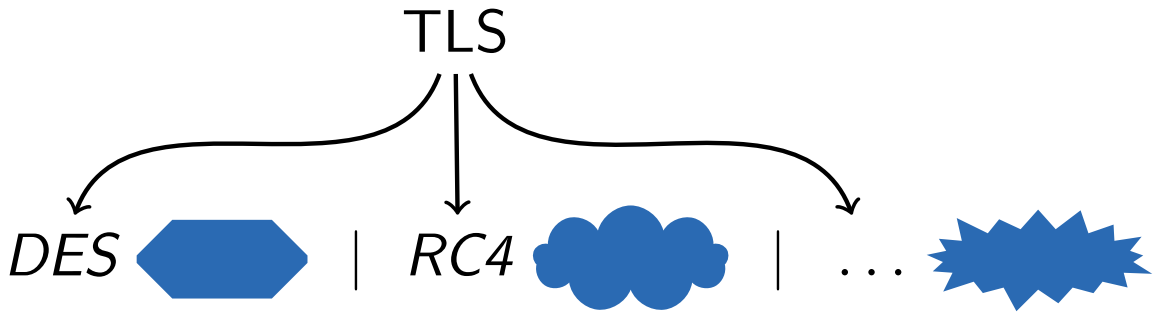

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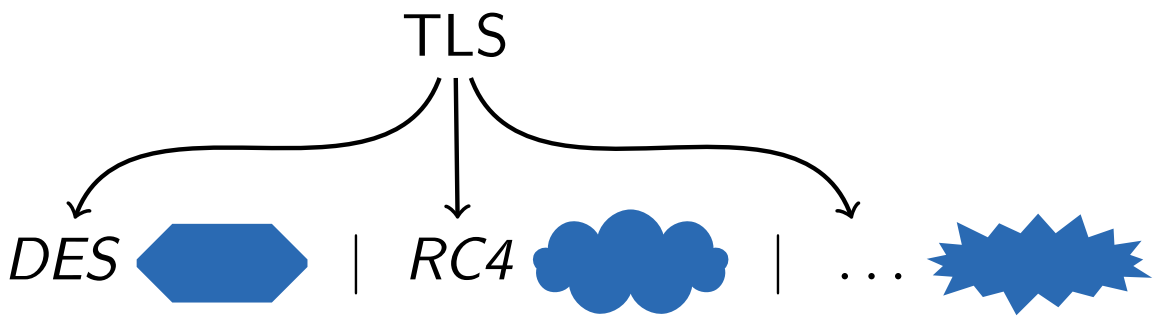

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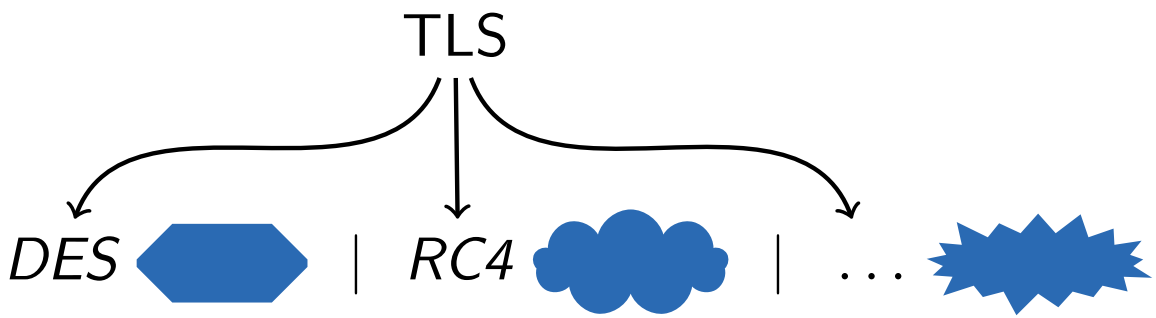

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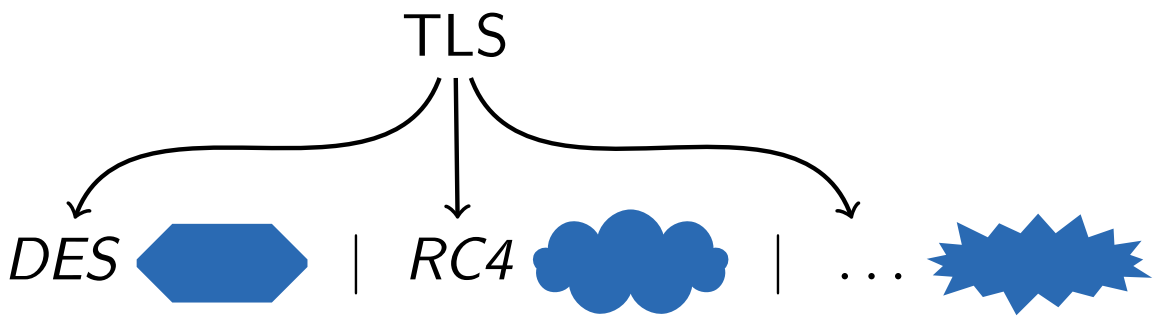
encrypt (*RC4 s*) *m* = *apfst RC4* (*encrypt-rc4 s m*)

difficult to add new ciphersuites

datatype σ *cipher* = *Cipher* σ (σ ⇒ *string* ⇒ σ × *string*)
encrypt (*Cipher s enc*) *m* = (λ(*s*′, *m*′). (*Cipher s*′ *enc*, *m*′)) (*enc s m*)

Existential state types

TLS state stores varying ciphersuites



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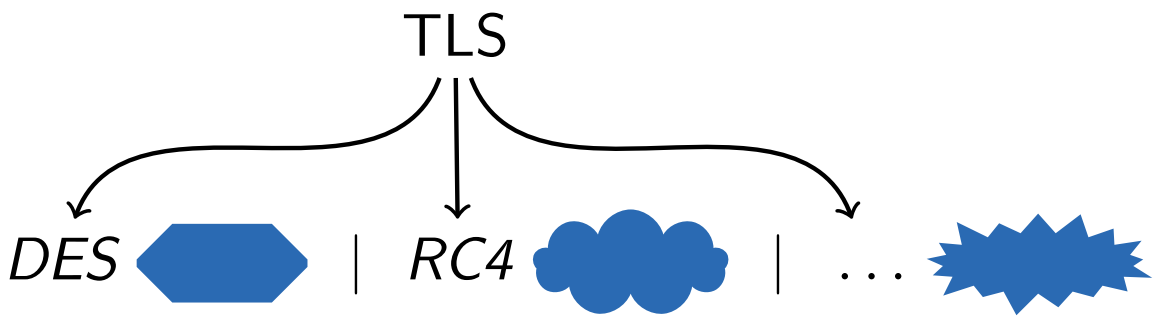

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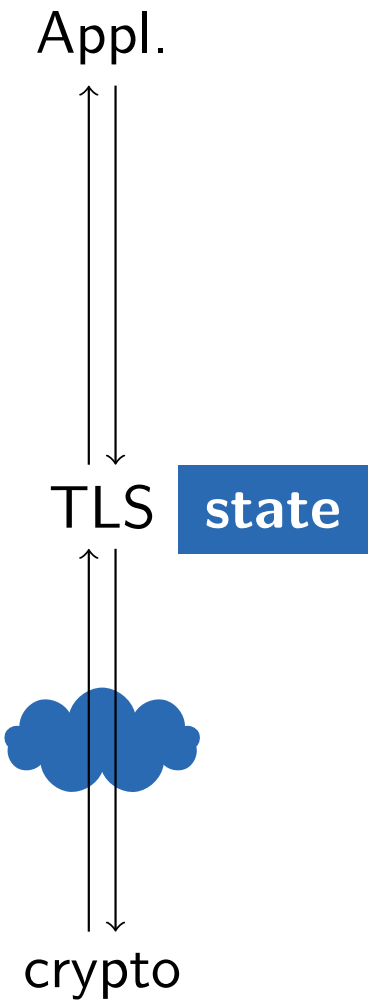
Avoid existential type by modelling the observations

codatatype *cipher* = *Cipher* (*encrypt*: *string* ⇒ *cipher* × *string*)

primcorec *mk-rc4* :: ⇒ *cipher* where
encrypt (*mk-rc4 s*) = *map-pair mk-rc4 id* ∘ *encrypt-rc4 s*

Dealing with state



State passing in the monad

- new state type variable σ in $(\alpha, o, \iota, \sigma)$ *resumption*

  hard to combine functions
  that store different kinds of things

- universal type of storable data
  Imperative HOL: countable first-order values

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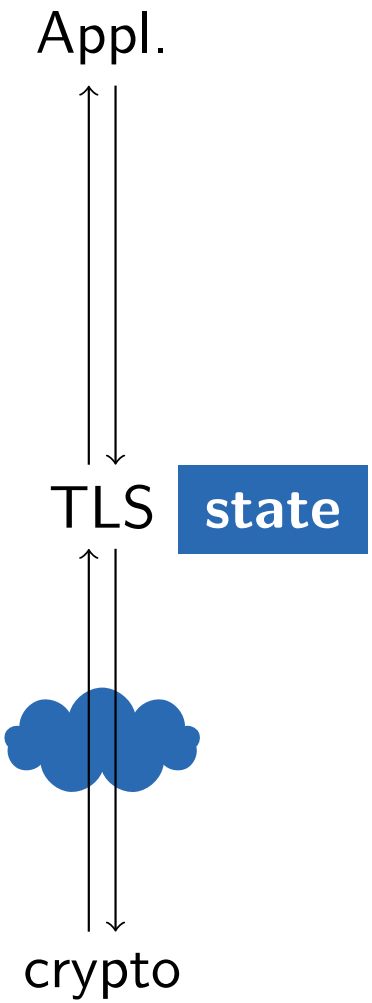

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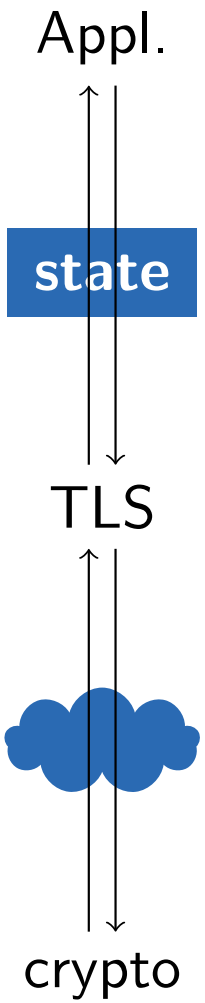

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It is feasible to program TLS in Isabelle/HOL

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Insights from the case study:

FFI imports **code-printing** offers basic functionality,
but types for data exchange are under-developed.

Unit tests instead of proofs,
but existing evaluation tools do not work with FFI imports.

Type system is very restrictive.

- monad transformers
- abstraction and information hiding

BNF The new (co)datatype package is great!