Stream Fusion for Isabelle's Code generator

Andreas Lochbihler Alexandra Maximova

Institute of Information Security
ETH Zurich, Switzerland

Interactive Theorem Proving 2015

From formal specifications to implementations

specification

implementation

Standard
ML

$$sum\text{-}primes\ n = sum\ (filter\ prime\ [1\ ..\ n])$$

From formal specifications to implementations

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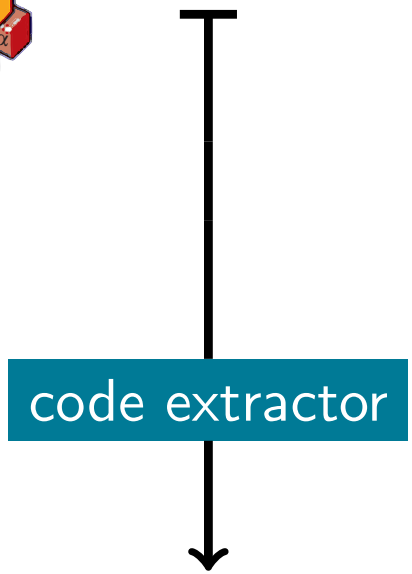

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fun sum xs    = ...
fun filter p  = ...
fun prime x   = ...
fun upt i j   = ...
fun sum_primes n =
    sum (filter prime (upt 1 n));
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From formal specifications to implementations

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Scala Standard ML Haskell

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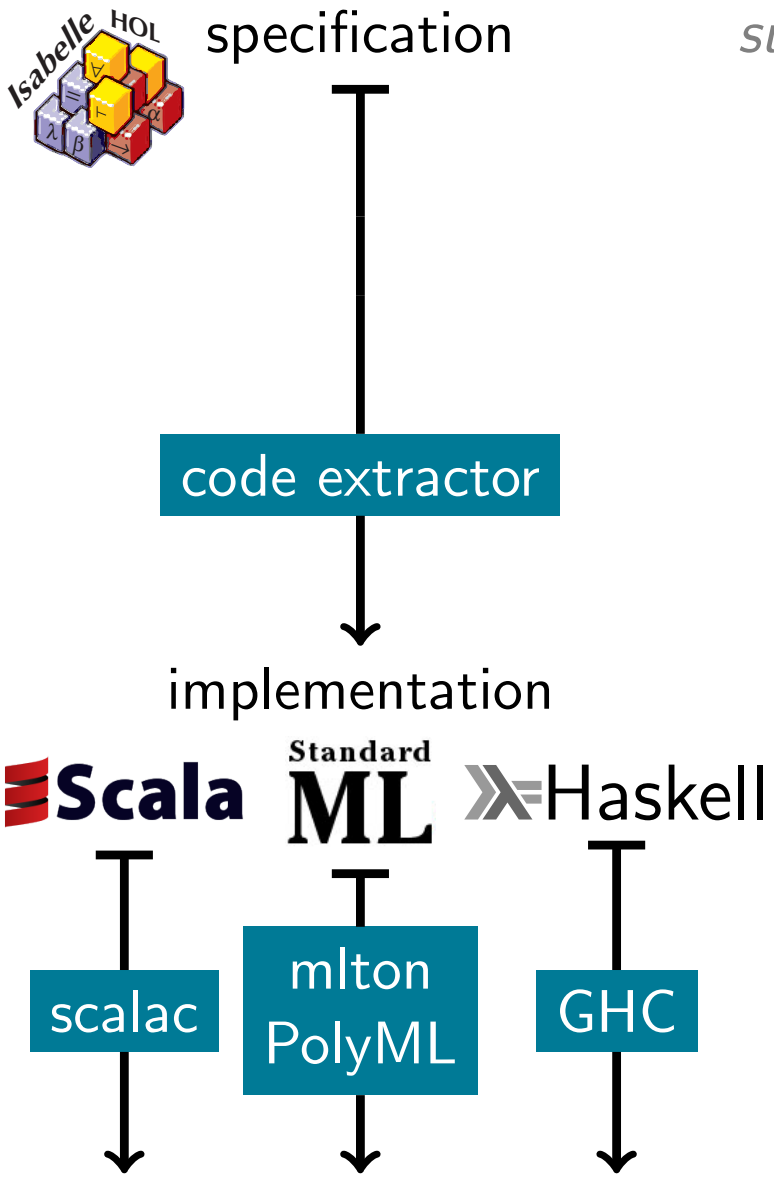

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scalac mlton PolyML GHC

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From formal specifications to implementations

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sum-primes n = *sum* (*filter prime* [1 .. *n*])

preprocessor

perform optimisations before extraction?

code extractor

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Scala | Standard ML | Haskell

- ⊕ more structure is available
- ⊕ more flexibility in transformations
- ⊕ exploit proven theorems
- ⊕ kernel checks the transformation

scalac | mlton PolyML | GHC

optimise for efficiency

From formal specifications to implementations

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sum-primes n = sum (filter prime [1 .. n])

preprocessor

perform optimisations before extraction?

Focus on enabling transformations

Contributions

1. Formalised stream fusion in HOL
2. Verified a library of list functions
3. Implemented the transformation in Isabelle's code generator

scalac

PolyML

GHC

efficiency

List fusion

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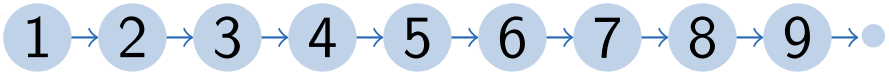
Evaluate sum-primes 9**:**

alloc

$[1\ ..\ 9]$ 

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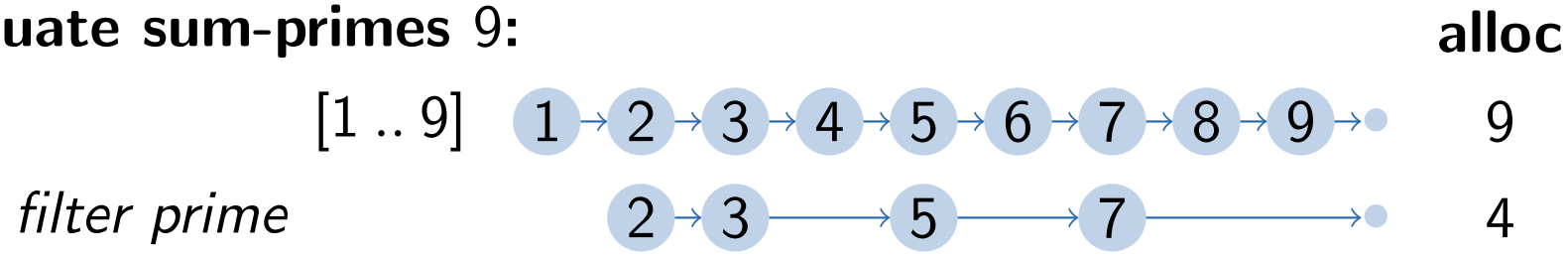

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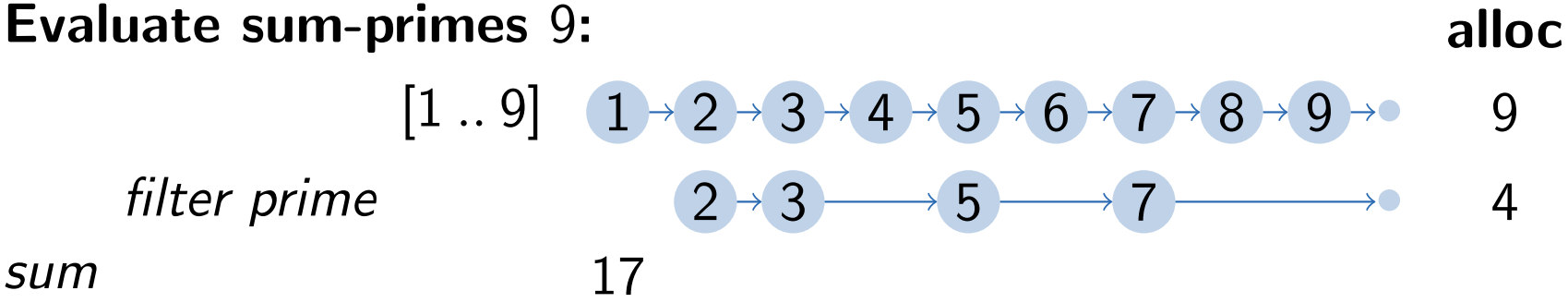

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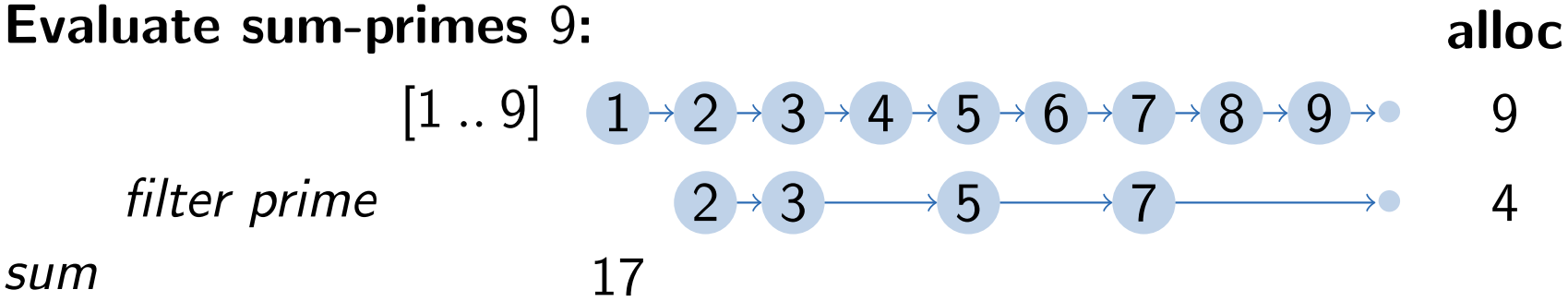

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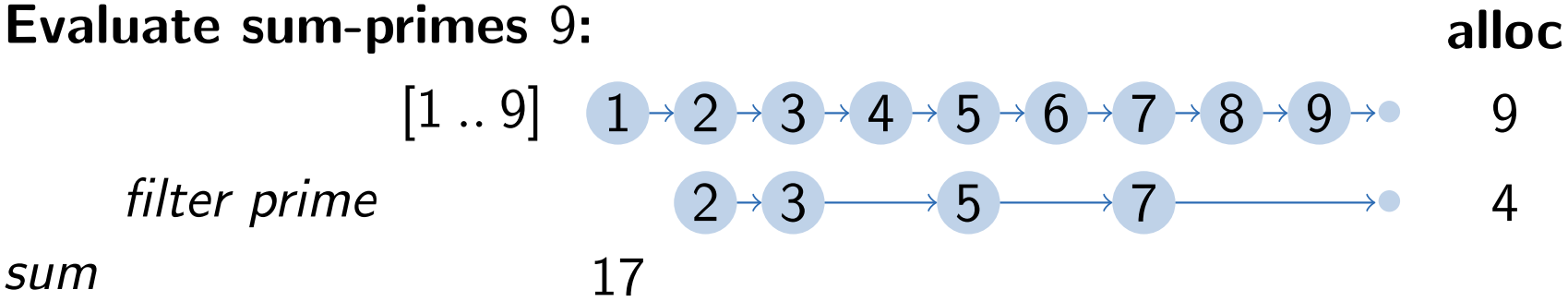

Compilers fail to eliminate intermediate lists
because *filter* and [_ .. _] are recursive.

Express list transformers and producers without recursion
such that they can be inlined into consumers.

Shortcut fusion [Svenningsson 2002]

datatype α *list* $=$
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 $\mid \alpha \cdot \alpha$ *list*

datatype $(\alpha,\ \sigma)$ *step* $=$
  Done
 $\mid$ *Yield* $\alpha\ \sigma$

type α *stream* $=$
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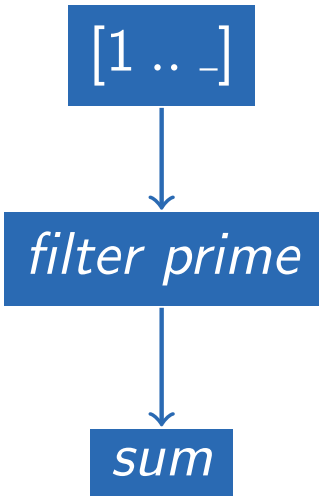

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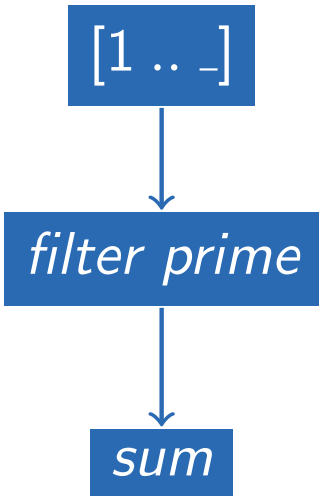

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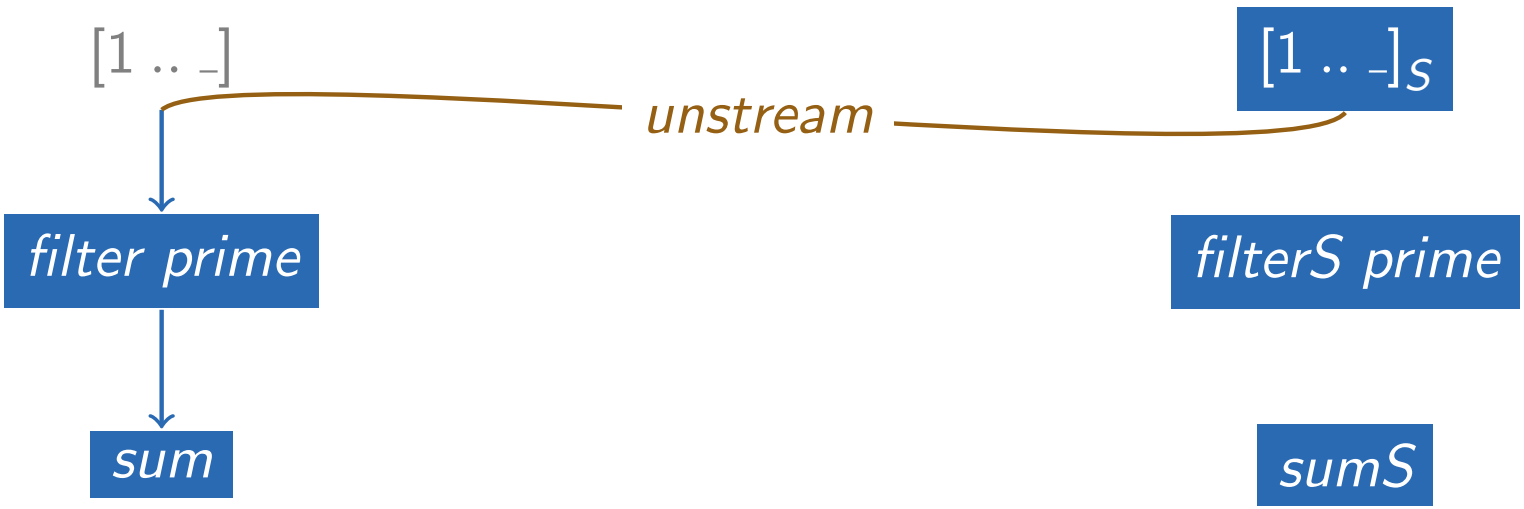

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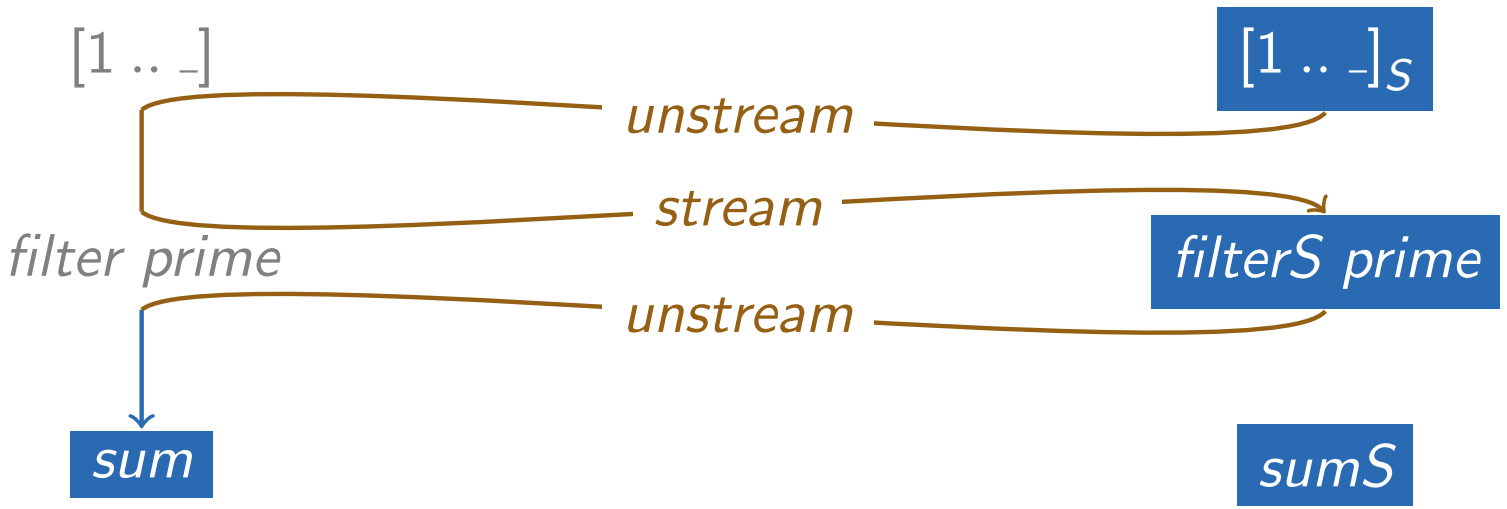

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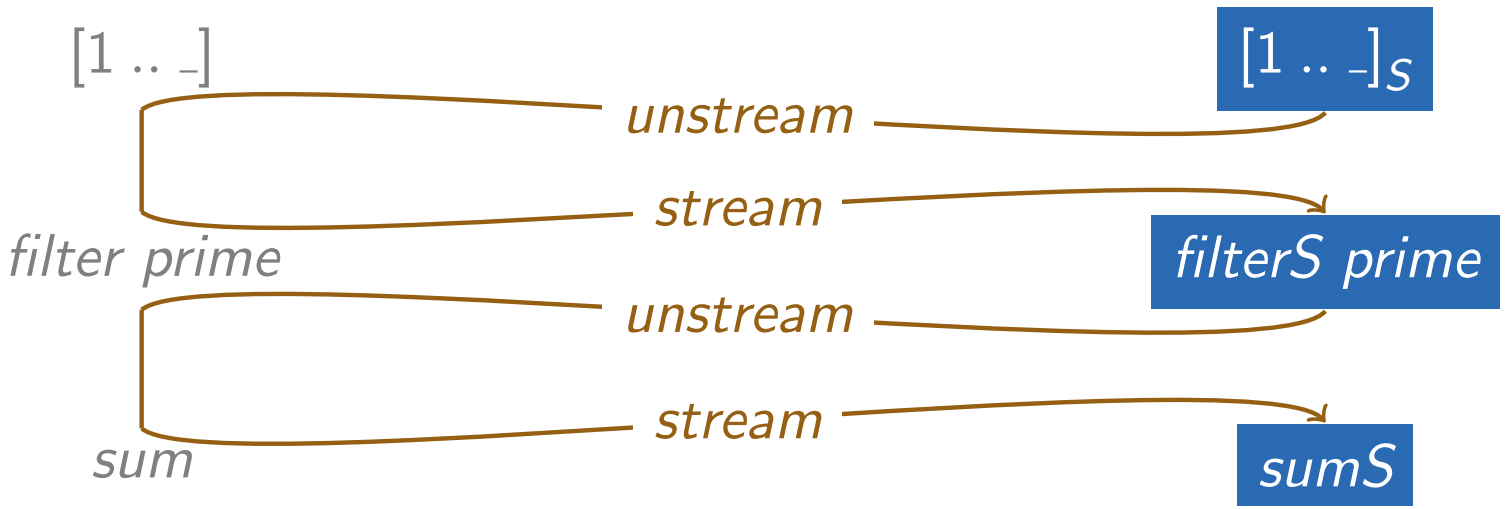

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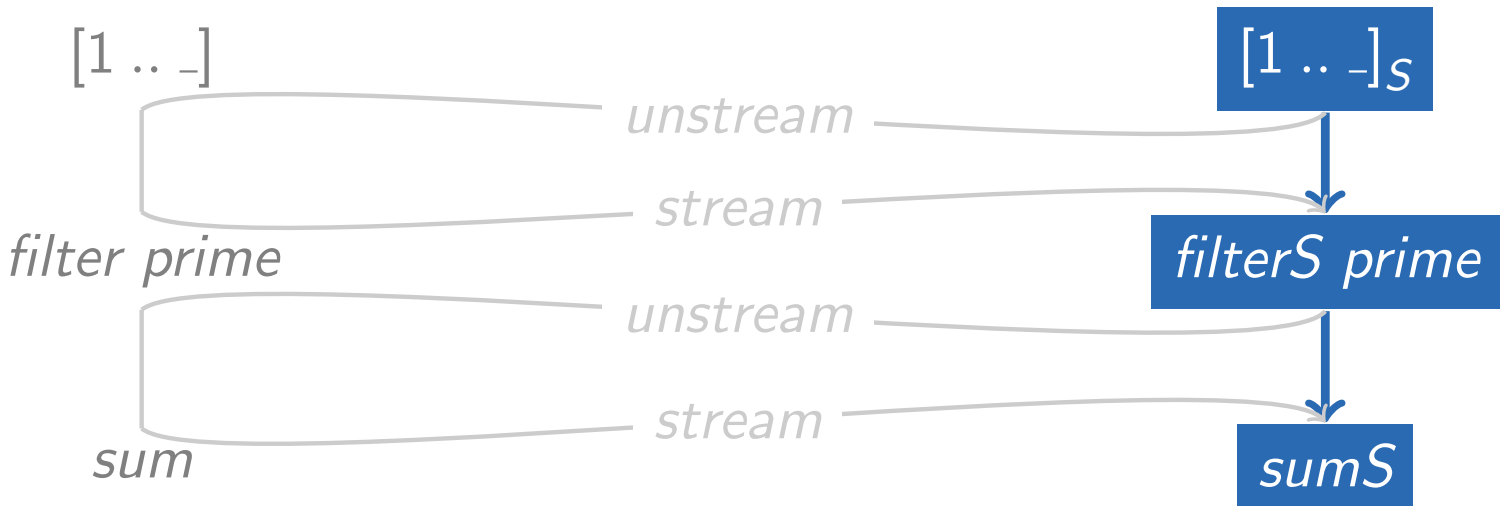

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Challenges in HOL

1. No existential types
   Fusion equation $\mathit{stream} \circ \mathit{unstream} = \mathit{id}$ does not type-check.

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2. *list* and *stream* are not isomorphic
   inductive lists — use subtype of **terminating** generators
   $\rightsquigarrow$ return *Done* eventually

   coinductive lists — use subtype of **productive** generators
   $\rightsquigarrow$ only finitely many consecutive skips

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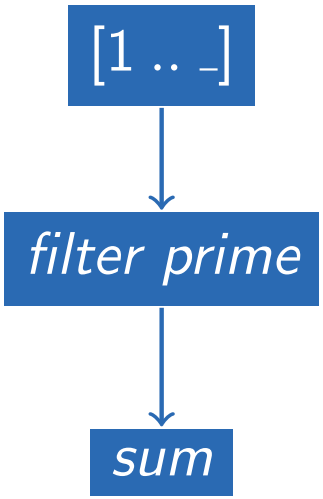

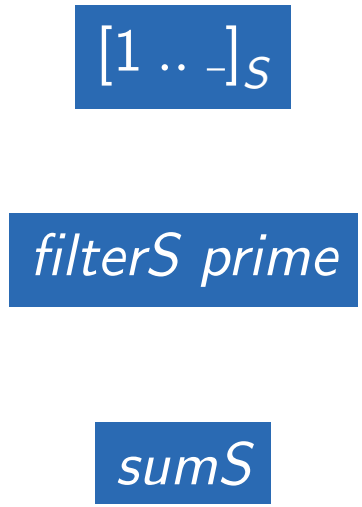

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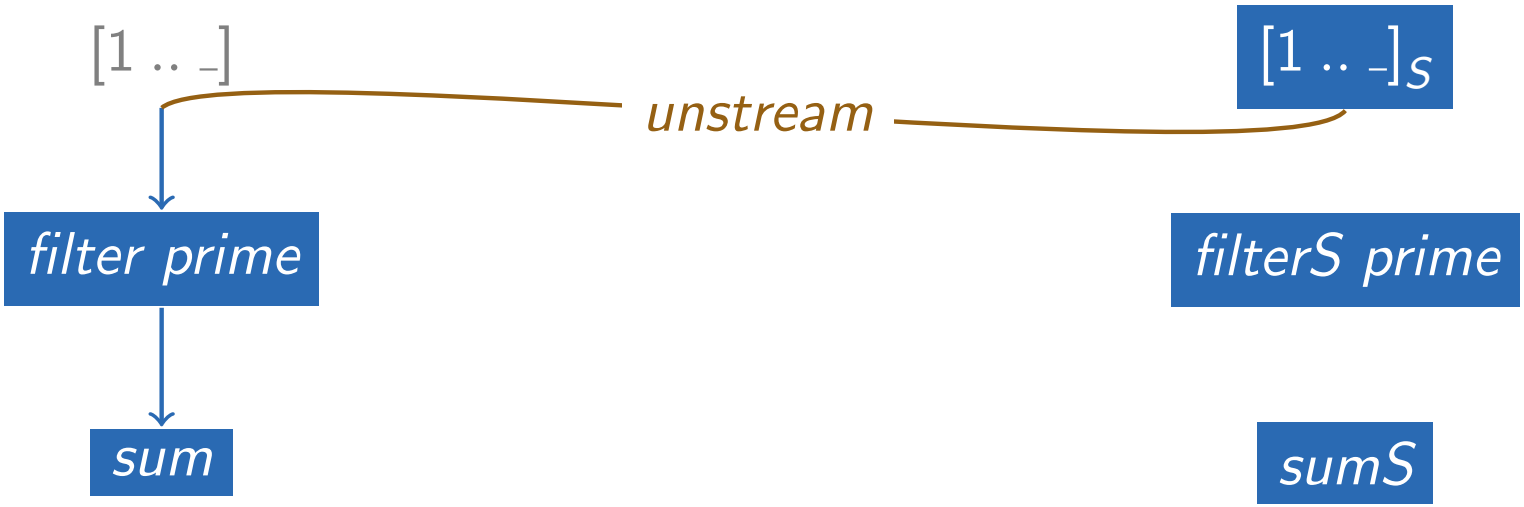

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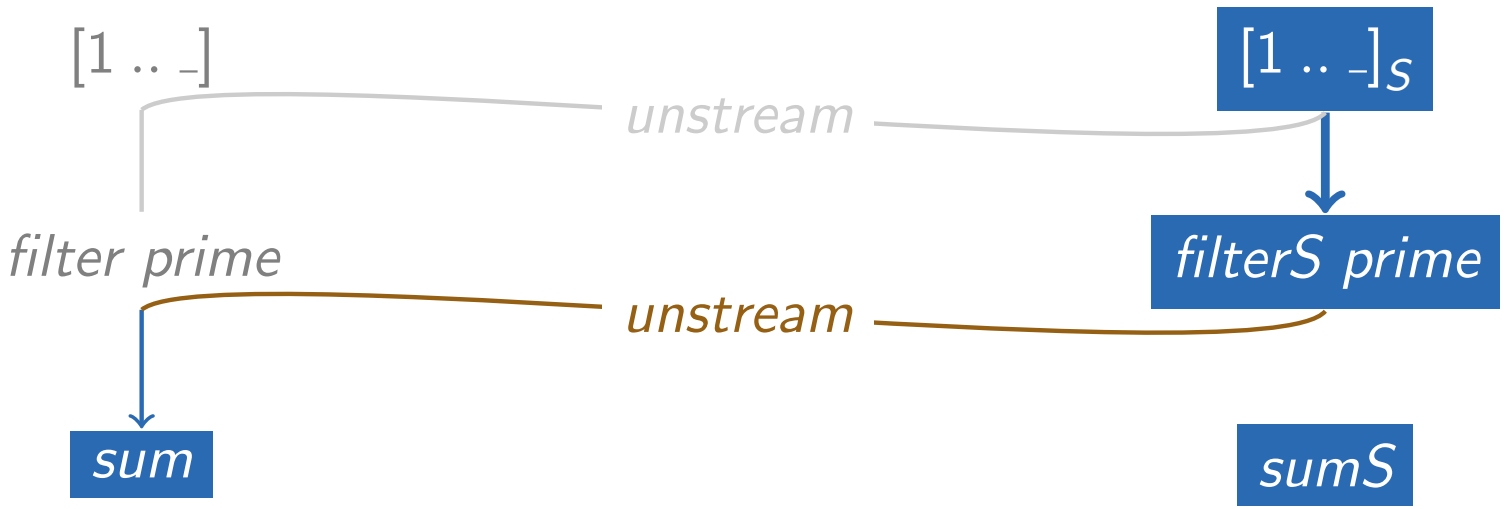

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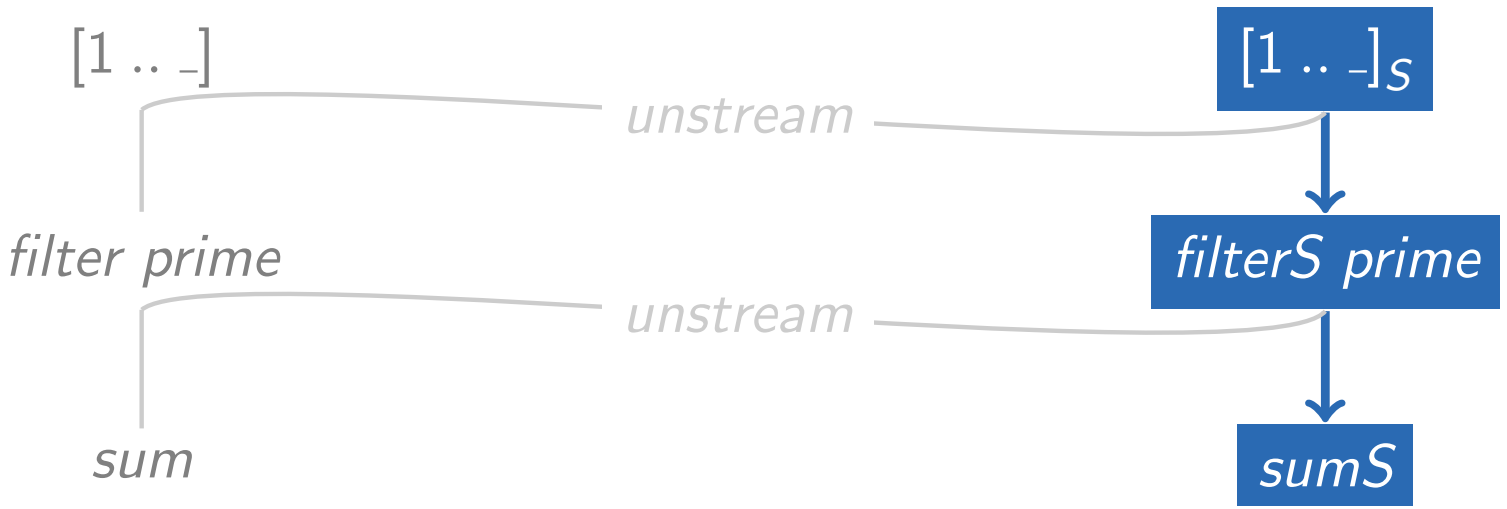

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Available in the Archive of Formal Proofs

`http://afp.sf.net/entries/Stream_Fusion_Code.shtml`