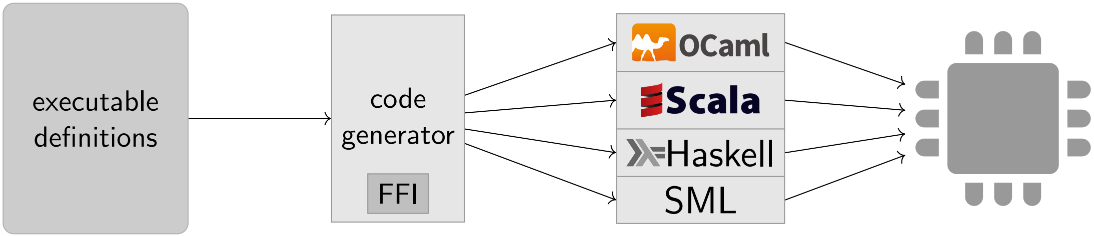
Fast machine words in

Andreas Lochbihler

Digital Asset (Switzerland) GmbH

Code generation in Isabelle HOL



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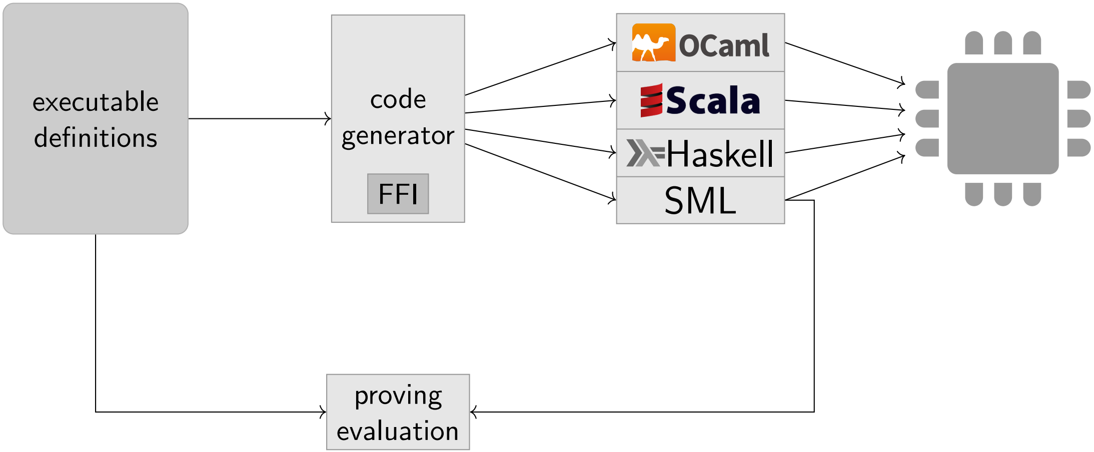

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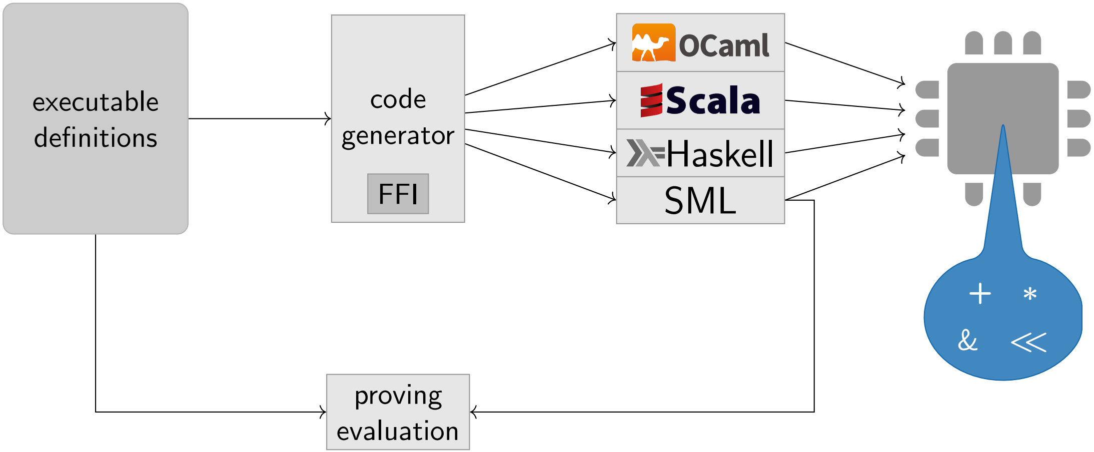

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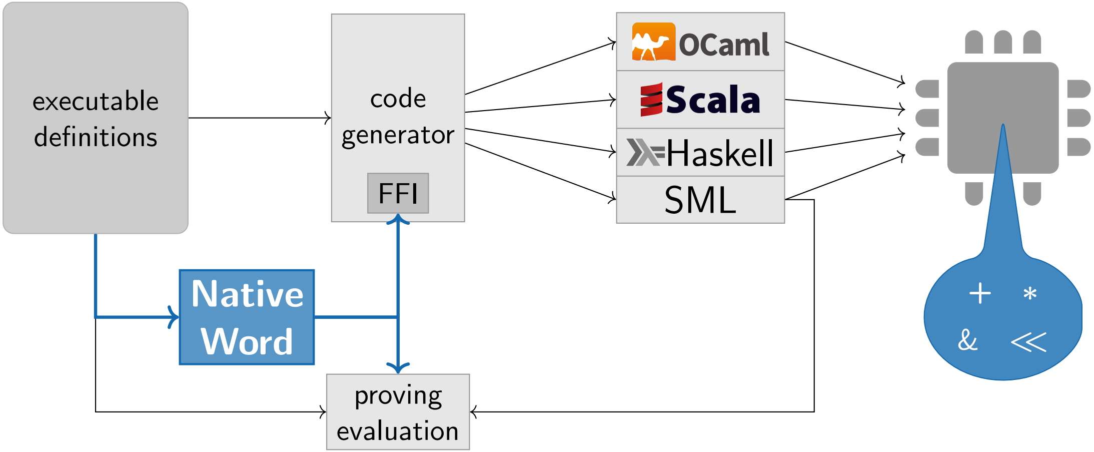

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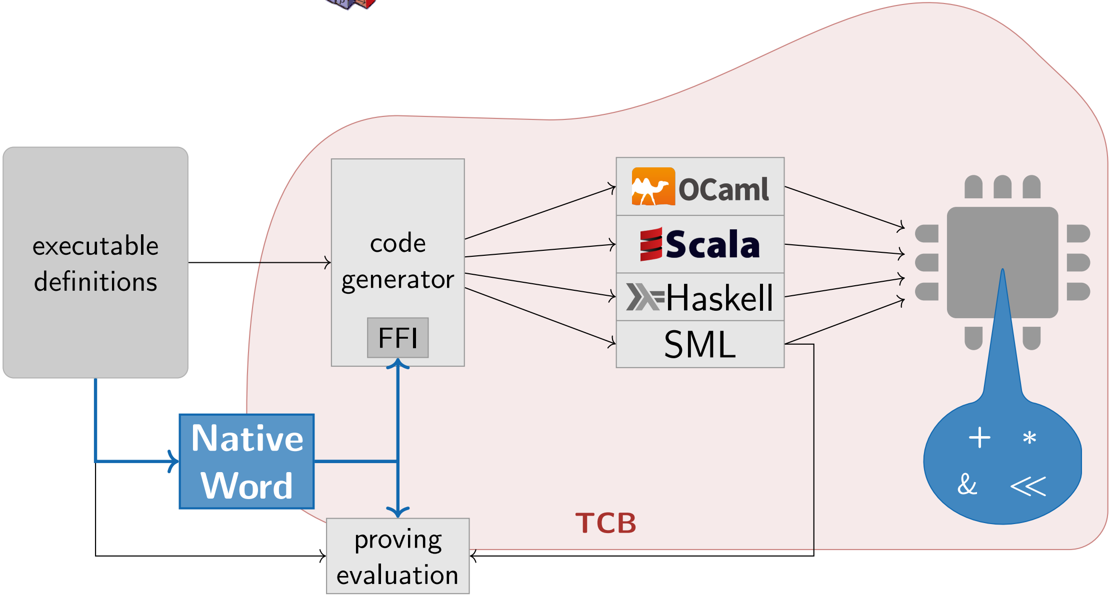

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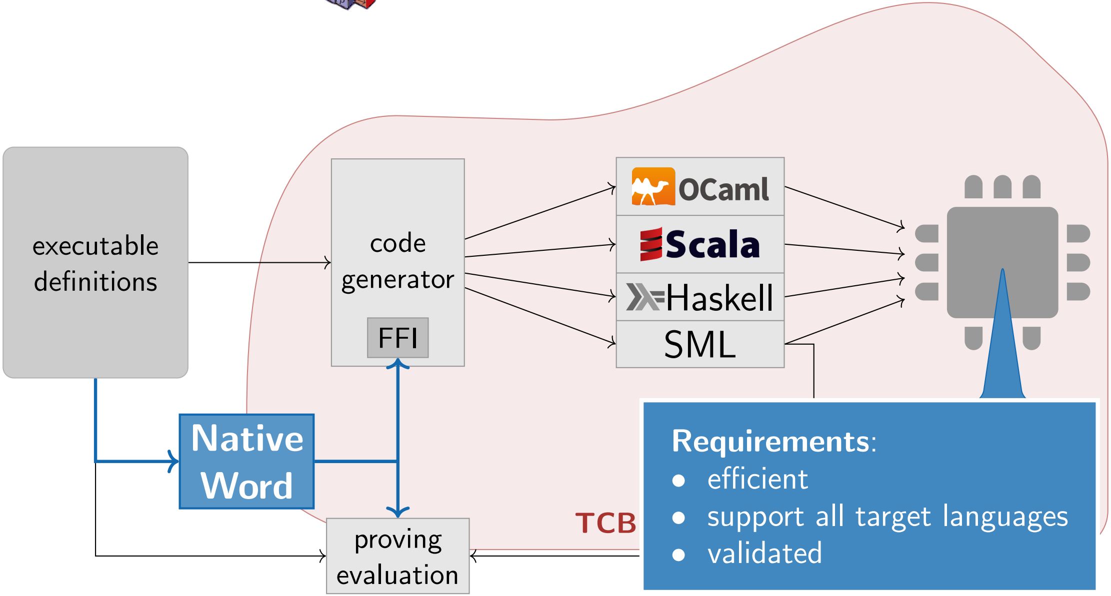
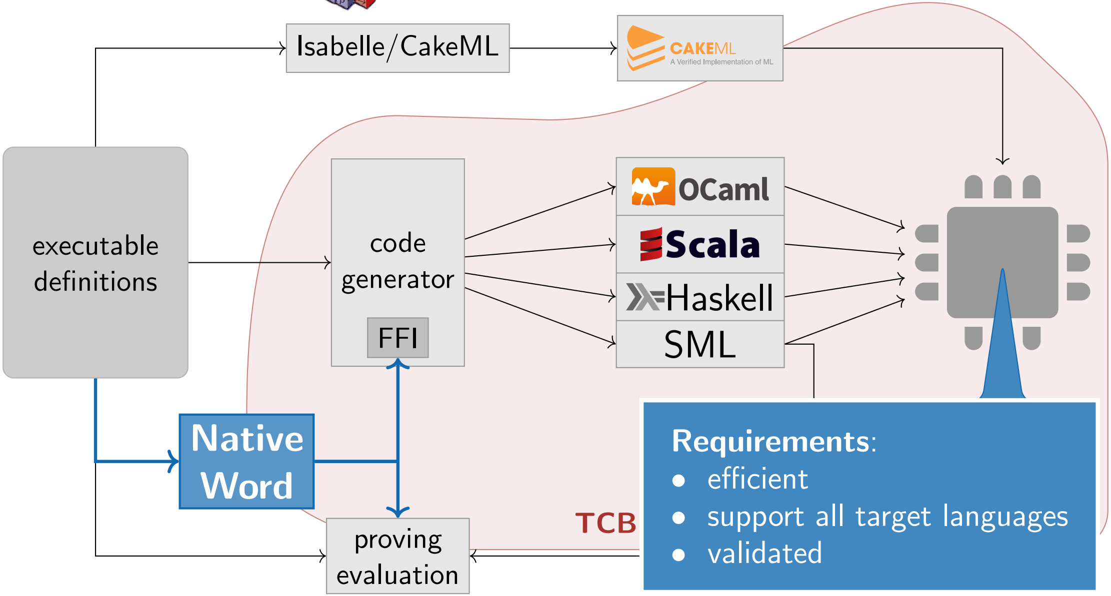


Requirements:
- efficient
- support all target languages
- validated

Code generation in Isabelle/HOL



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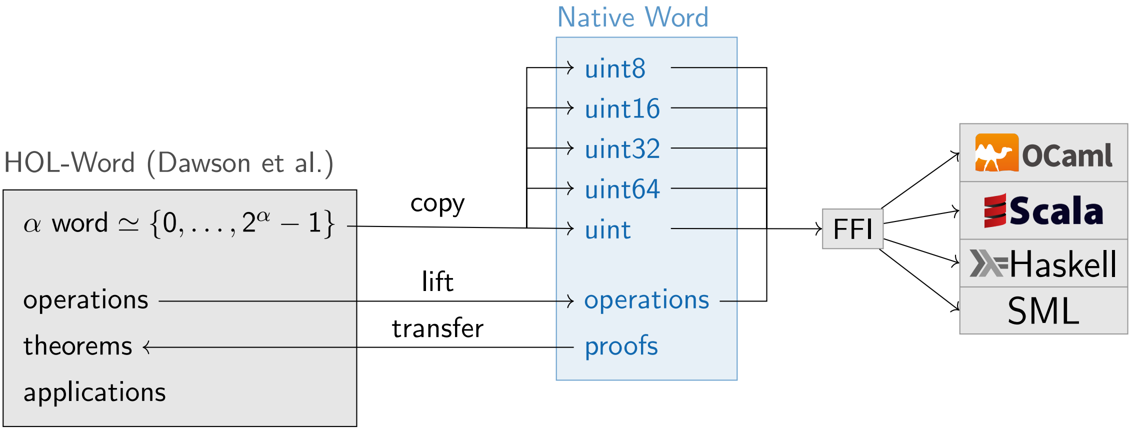
Available bit-widths

bits	PolyML		SMLNJ	mlton	OCaml		GHC	Scala
	32	64			32	64		
8	√	√	√	√			√	√
16				√			√	√
32	√	√	√	√	√	√	√	√
64		√	√	√	√	√	√	√
default	31	63	31	32	31	63	≥ 30	32

(grey shading) = signed operations only

Let's abstract over these differences I



Let's abstract over these differences II

Conventional approach

1. Identify subset of common behaviour

```
definition divmod-abs x y =
  (|x| div |y| , |x| mod |y|)
```

2. Reduce to restricted behaviour

```
lemma [code]: divmod x y =
  ... if sgn x = sgn y then divmod-abs x y
      else ...
```

3. Common FFI for all languages

```
code-printing divmod-abs →
  (Haskell) divMod (abs _) (abs _)
  (OCaml) ...
  (Scala) ...
  (SML) ...
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2 case distinctions on the sign of each operand

PolyML: 2X slowdown

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Cascading

1. Model behaviours of target languages

```
definition uint32-div x y = ...
definition uint32-sdiv x y = ...
```

2. Build cascade of models

```
lemma [code]:
  div x y = ... uint32-div ...
  uint32-div x y = ... uint32-sdiv ...
```

3. One FFI for each language

```
code-printing uint32-div →
  (Haskell) Prelude.div
code-printing uint32-sdiv →
  (OCaml) Int32.div
code-printing ... → ...
```

Let's abstract over these differences II

Conventional approach

1. Identify subset of common behaviour

```
definition divmod-abs x y =
  (|x| div |y| , |x| mod |y|)
```

2. Reduce to restricted behaviour

```
lemma [code]: divmod x y =
  ... if sgn x = [illegible]
      else ...
```

3. Common FFI for all languages

```
code-printing divmod-abs →
  (Haskell) divMod (abs _) (abs _)
  (OCaml) ...
  (Scala) ...
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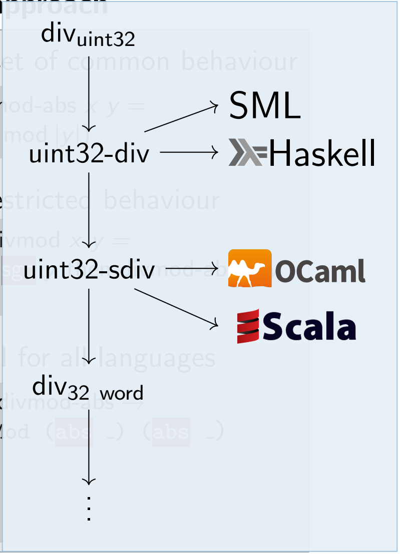

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code-printing ... → ...
```

What about unspecified behaviour?

Underspecification in OCaml

x << n is **undefined**
if $n > 32$

← `code-printing`

Underspecification in HOL

`definition` uint32-shiftl x n =
  if $n \leq 32$ then $x \ll n$
  else **undefined** $(\ll)$ x n

`lemma` [*code*]: $x \ll n =$
  if $n \leq 32$ then uint32-shiftl x n else 0

Underspecification leads to refinement

HOL axioms
definitions

Correctness w/o underspecification:
If code c terminates with result r,
then we can derive c = r.

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Every derivable property of the code c
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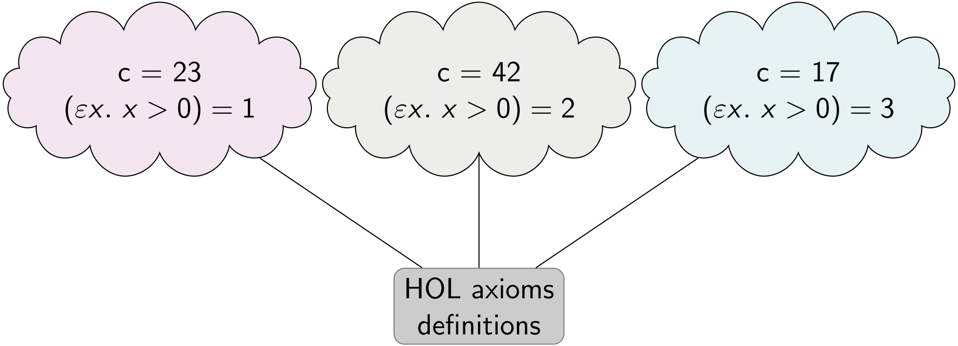

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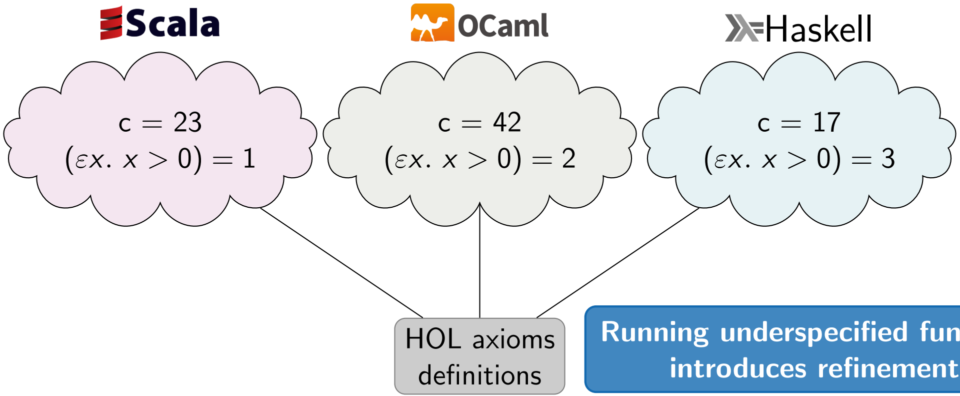

Running underspecified functions introduces refinement!

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Every derivable property of the code c applies to the result r.

Underspecification leads to refinement

Scala

$c = 23$

$(\varepsilon x.\ x > 0) = 1$

OCaml

$c = 42$

$(\varepsilon x.\ x > 0) = 2$

Haskell

$c = 17$

$(\varepsilon x.\ x > 0) = 3$

HOL axioms definitions

Forbid underspecification for proofs!

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Every derivable property of the code c applies to the result r.

Default word size with underspecified bit width

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64		√	√	√	√	√	√	√
default	**31**	**63**	**31**	**32**	**31**	**63**	$>$ **30**	**32**

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Unspecified bit size

- hashing
- bit vectors
- dynamic implementation choices based on input size

Validation

- Framework to run test cases from within Isabelle/HOL

  `test-code` $\;251\ \mathsf{div}\ 3 = 83\;$ `in` Scala

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  `test-code` $251 \operatorname{div} 3 = 83$ `in` Scala SMLNJ MLton GHC PolyML

- Test cases for all operations on uint$*$

Validation

- Framework to run test cases from within Isabelle/HOL

  `test-code` $251 \operatorname{div} 3 = 83$ **in** Scala SMLNJ MLton GHC PolyML

- Test cases for all operations on uint$*$

- Revealed many errors in the FFI mapping – now fixed
- Found one error in PolyML 5.6 in 64-bit mode – fixed in 5.7

  $18446744073709551611 \operatorname{div} 3$ evaluates to 1431655763

Usage and Benchmarks

Usage:

- IsaFoR (Berlekamp-Zassenhaus)
- Fleury's verified SAT solver
- CAVA model checker
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Factor 400 polynomials over $\mathbb{Z}/p^k\mathbb{Z}$

Strategies:

1. Use unbounded GMP integers int.
2. If $p^k < 2^{16}$ use uint32.
   If $p^k < 2^{32}$ use uint64.
   Else use int
3. If $p^k < 2^{\text{default}/2}$ use uint.
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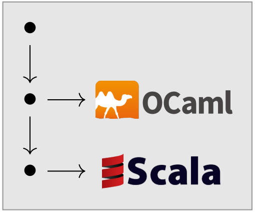
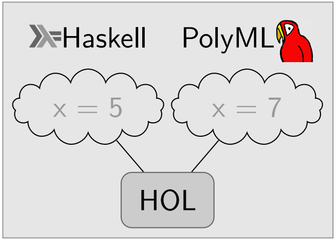
GHC	2 is **18 % faster** than 1.
PolyML	3 is **4 % faster** than 2.

Takeaways

1. Cascade pattern



2. Model-theoretic underspecification



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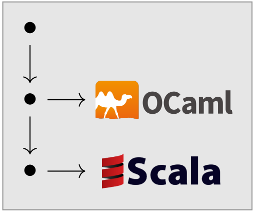

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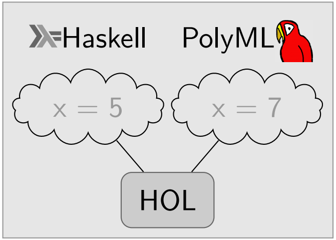

Try it out!

Native Word

in the Archive of Formal Proofs

www.isa-afp.org/entries/Native_Word.html

Testing framework

in the Isabelle distribution

HOL-Library.Code_Test